Anomaly Mediation in Superstring Theory

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ISM 2011, Puri

Based on papers 1007.5145 and 1008.4361 with Mark Goodsell and Eran Palti

Definitions and Motivation   **Motivations**
Calculations   Relationship to Gauge Couplings
Conclusions   The Model

Motivations

Supersymmetry broken at the TeV scale is one of the main targets of the LHC program.

The breaking is described by

$$\mathcal{L} = \mathcal{L}_{\text{Supersymmetric Standard Model}} + \mathcal{L}_{\text{soft terms}}$$

The soft terms are *relevant terms* in the Lagrangian that *explicitly break* supersymmetry but *do not reintroduce* quadratic divergences

They consist of:

Scalar masses $m^2|\phi^2|$,   Trilinear scalar A-terms $A_{\alpha\beta\gamma}\phi^\alpha\phi^\beta\phi^\gamma$

Gaugino masses $M_a\lambda\lambda$,   Bilinear scalar B-terms $B_{ab}\phi^a\phi^b$.

Motivations

Models of supersymmetry breaking are primarily about computing the soft terms.

There are two main options for supersymmetry breaking :

- ▶ Gauge mediation - supersymmetry breaking is generated within global field theory, $m_{soft} \gg m_{3/2}$.
- ▶ Gravity mediation - supersymmetry breaking arises from Planck-suppressed operators, $m_{soft} \lesssim m_{3/2}$.

In gravity mediation, susy breaking comes from operators generated at the string scale.

The structure of supersymmetric soft terms may be a direct insight into the UV theory and it is interesting to study susy breaking within string theory.

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Motivations

The structure of supersymmetry breaking is determined by the effective 4d supergravity theory for moduli (Φ) and matter (C^α).

$$
\begin{aligned}
W &= \hat{W}(\Phi) + \mu(\Phi)C^\alpha C^\beta + \frac{1}{6}Y_{\alpha\beta\gamma}(\Phi)C^\alpha C^\beta C^\gamma + \dots, \\
K &= \hat{K}(\Phi, \bar{\Phi}) + \tilde{K}_{\alpha\bar{\beta}}(\Phi, \bar{\Phi})C^\alpha C^{\bar{\beta}} + \left[ZC^\alpha C^\beta + h.c. \right] + \dots, \\
f_a &= f_a(\Phi). \\
V &= e^{\hat{K}}(\hat{K}^{i\bar{j}}D_i\hat{W}D_{\bar{j}}\bar{\hat{W}} - 3|\hat{W}|^2).
\end{aligned}
$$

Classical soft terms come from expanding the $\mathcal{N} = 1$ supergravity action in terms of the moduli and matter fields.

If these vanish loop level soft terms become important.

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Definition

This talk is about computing anomaly-mediated gaugino masses directly in string theory.

'Anomaly-mediated' will mean any mass term of generic 1-loop order

$$M_{\lambda,anomaly} \sim \frac{g^2 m_{3/2}}{16\pi^2}$$

Comparison:

$$M_{\lambda,tree} : \mathcal{O}(m_{3/2})$$

$$M_{\lambda,running} : \mathcal{O}(m_{3/2})\frac{g^2}{16\pi^2} \ln\left(\frac{\Lambda^2}{\mu^2}\right)$$

Gaugino Masses and Gauge Couplings

Supersymmetry relates gaugino masses and gauge couplings

$$
\int d^4x d^2\theta \, f(\Phi) W_\alpha W^\alpha \;=\; \int d^4x \, \mathrm{Re}(f(\Phi)) F_{\mu\nu} F^{\mu\nu}
$$
$$
+ \int d^4x \, F_{f(\Phi)} \lambda\lambda + \dots
$$

Anomalous gaugino masses are closely related to gauge threshold corrections.

Gauge threshold corrections (anomalous gauge couplings) have been well studied in string theory.

Anomalous Gauge Couplings

In supergravity, physical and holomorphic gauge couplings are related by Kaplunovsky-Louis formula:

$$g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) = \qquad \text{Re}(f_a(\Phi)) \qquad \text{(Holomorphic coupling)}$$

$$+ \frac{b_a}{16\pi^2} \ln\left(\frac{M_P^2}{\mu^2}\right) \qquad (\beta\text{-function running})$$

$$+ \frac{T(G)}{8\pi^2} \ln g_{phys}^{-2}(\Phi, \bar{\Phi}, \mu) \qquad \text{(NSVZ term)}$$

$$+ \frac{(\sum_r n_r T_a(r) - T(G))}{16\pi^2} \hat{K}(\Phi, \bar{\Phi}) \qquad \text{(Kähler-Weyl anomaly)}$$

$$- \sum_r \frac{T_a(r)}{8\pi^2} \ln \det Z^r(\Phi, \bar{\Phi}, \mu). \qquad \text{(Konishi anomaly)}$$

Relates *measurable* couplings and *holomorphic* couplings.

This expression is well studied within string theory via gauge threshold corrections.

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Anomalous Gaugino Masses

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[-\left(T_G - T_R\right)K_i F^i - \frac{2T_R}{d_R}F^i\partial_i\left(\ln\det Z\right) + 2T_G F^I\partial_I\ln\left(\frac{1}{g_0^2}\right)\right]$$

(*Kaplunovsky, Louis*)(*deAlwis*)

$$m_{1/2} = -\frac{g_a^2}{16\pi^2}\left[3T(G) - T(R)\right]m_{3/2} + (\ldots)$$

(*Giudice, Luty, Murayama, Rattazzi*), (*Randall, Sundrum*)

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[\left(3T_G - T_R\right)m_{3/2} - \left(T_G - T_R\right)K_i F^i - \frac{2T_R}{d_R}F^i\partial_i\left(\ln\det Z\right)\right]$$

(*Bagger, Moroi, Poppitz*), (*Gaillard, Nelson*)

$$m_{1/2} = -\frac{g^2}{16\pi^2}\left[\left(3T_G - T_R\right)m_{3/2} - \left(T_G - T_R\right)K_i F^i - \frac{2T_R}{d_R}F^i\partial_i\left(\ln\det Z\right)\right.$$
$$\left. + 2T_G F^I\partial_I\ln\left(\frac{1}{g_0^2}\right)\right] .$$

Also see (*Dine, Seiberg*).

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Motivations

$$m_{1/2} = -\frac{g^2}{16\pi^2} \left[(3T_G - T_R) \, m_{3/2} - (T_G - T_R) \, K_i F^i - \frac{2T_R}{d_R} F^i \partial_i (\ln \det Z) \right.$$
$$\left. + 2T_G F^I \partial_I \ln \left(\frac{1}{g_0^2} \right) \right] \, .$$

Why study in string theory?

▶ Expression is ultraviolet sensitive (depends on K and Z)

▶ Expression is classically undetermined (not invariant under $K \to K + h + \bar{h}$, $W \to e^h W$).

▶ Not complete agreement about precise form of expression (in particular $m_{3/2}$ term)

▶ Previous stringy argument for no anomalous gaugino masses
  (Antoniadis, Taylor)

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Motivations

Why study in string theory?

In no-scale susy breaking models (occurs in e.g. LARGE volume scenario)

$$
\begin{aligned}
K &= -3\ln(T + \bar{T}) + \frac{Q\bar{Q}}{T + \bar{T}}, \\
W &= W_0,
\end{aligned}
$$

there is a cancellation of anomaly-mediated terms

$$
m_{1/2} = -\frac{g^2}{16\pi^2}\left[(3T_G - T_R)\, m_{3/2} - (T_G - T_R)\, K_i F^i - \frac{2T_R}{d_R} F^i \partial_i \left(\ln \det Z\right)\right] = 0.
$$

The existence of this cancellation is phenomenologically important for the magnitude and structure of soft terms.

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Where to compute?

We want examples of calculable models with non-zero beta functions.

- ▶ The simplest such examples are (fractional) D3 branes at orbifold singularities.

- ▶ String can be exactly quantised and all calculations can be performed explicitly.

- ▶ Orbifold singularities only involve annulus amplitudes further simplifying the computations.

- ▶ Will focus on D-branes at $\mathbb{C}^3/\mathbb{Z}_4$.

- ▶ Orbifold action is $(z_1, z_2, z_3) \rightarrow (\omega z_1, \omega z_2, \omega^2 z_3)$ with $\omega = e^{2\pi i/4}$.

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- ► The quiver for $\mathbb{C}^3/\mathbb{Z}_4$ is:
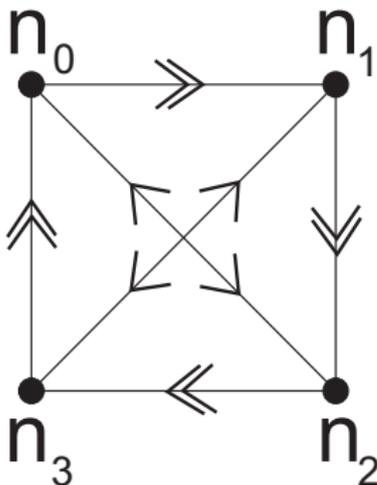
- ► Anomaly cancellation requires $n_0 = n_2$, $n_1 = n_3$.
- ► This sources a twisted tadpole in the $\mathcal{N} = 2$ sector set by $n_0 - n_1$.

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Can embed the model in a compact space:
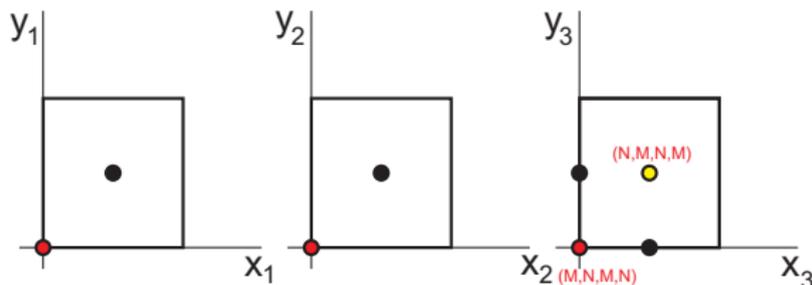
Kähler potential is given by

$$K = -\ln(S + \bar{S}) - \sum_{I=1}^{3} \ln\left((T_I + \bar{T}_I)(U_I + \bar{U}_I) - \frac{1}{6}(\Phi_I + \bar{\Phi}_I)^2 \right) .$$

$$K_{\Phi_I \bar{\Phi}_{\bar{I}}} \equiv Z_I = \frac{1}{(T_I + \bar{T}_I)(U_I + \bar{U}_I)} .$$

Twisted tadpole cancellation requires extra brane stacks on other orbifold fixed points.

Caveat: Uncancelled D3 tadpole in N=4 sector, should not be relevant for questions involving β-functions.

The Model
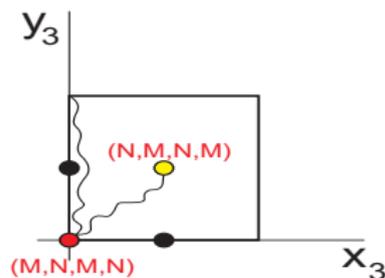
An important quantity is the winding mode partition function:

$$Z_{winding}(t) = \text{Tr}_{CP}(\Theta^2) \left(\sum_{n,m} e^{-(n^2 R_1^2 + m^2 R_2^2)\alpha' t} - \sum_{n,m} e^{-((n+\frac{1}{2})^2 R_1^2 + (m+\frac{1}{2})^2 R_2^2)\alpha' t} \right).$$

This satisfies

$$Z(t) \to 0 + \mathcal{O}(e^{-1/R^2 t}) \text{ as } t \to 0 \text{ (tadpole cancellation)}$$

$$Z(t) \to b_a \text{ as } t \to \infty \text{ (IR beta function)}$$

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Computations

Have computed the following amplitudes:

- $\langle A_\mu^a A_\nu^a \rangle, \langle \lambda^a \lambda^a \rangle$ — gauge threshold corrections

- $\langle \psi_i \psi_j \phi_k \rangle$ — Yukawa threshold corrections

- $\langle \phi_i \phi_j \phi_k \lambda_a \lambda_a \rangle$ — Tree-level brane-to-brane susy breaking

- $\langle H_3 \lambda_a \lambda_a \rangle$ — Anomalous gaugino masses from NS-NS fluxes

- $\langle F_3 \lambda_a \lambda_a \rangle$ — Anomalous gaugino masses from RR fluxes

In general, need to compute with off-shell momenta and go on-shell only at the end of the computation (to account for finite $\frac{0}{0}$ terms such as $\frac{k_i \cdot k_j}{k_i \cdot k_j}$).

This gives potential ambiguities for 3-point functions which can be resolved by considering 4-point functions.

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Gauge Thresholds
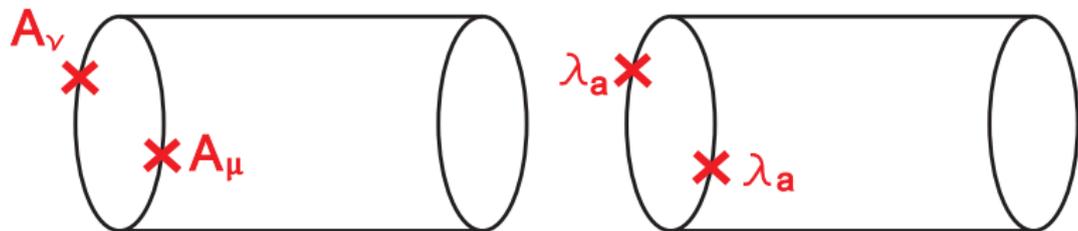
$$\mathcal{A} \sim \int \frac{dt}{t} \text{Tr}(\Theta^2) Z_{winding}(t)$$

$$\frac{1}{g^2}(\mu) = \frac{1}{g_0^2} + \frac{b_a}{16\pi^2} \ln\left(\frac{(RM_s)^2}{\mu^2}\right)$$

Same as previous results for these models (JC, Palti)- couplings run from the winding scale $M_W = RM_s$.

1-loop Yukawa Couplings
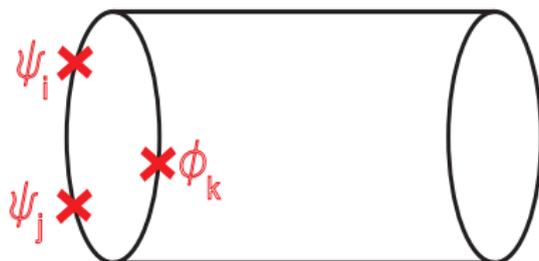
Basic vertex operators

$$
\begin{aligned}
\mathcal{V}^a_{-\frac{1}{2}}(u_1, k_1, z_1) &= t^a e^{-\phi/2} S^{\pm}(z_1) e^{ik_1 \cdot X(z_1)} e^{iq_1 \cdot H(z_1)}, \\
\mathcal{V}^b_{-\frac{1}{2}}(u_2, k_2, z_2) &= t^b e^{-\phi/2} S^{\pm}(z_2) e^{ik_2 \cdot X(z_2)} e^{iq_2 \cdot H(z_2)}, \\
\mathcal{V}^c_{-1}(u_3, k_3, z_3) &= t^c e^{-\phi} e^{ik_3 \cdot X(z_3)} e^{iq_3 \cdot H(z_3)}.
\end{aligned}
$$

Amplitude is

$$
\mathcal{A} = \int \frac{dt}{t} \int dz_1 dz_2 dz_3 \left\langle V^a_{-1/2}(u_1, k_1, z_1) \mathcal{V}^b_{\frac{1}{2}}(u_2, k_2, z_2) V^c_0(\varphi, k_3, z_3) \right\rangle.
$$

1-loop Yukawa Couplings

There are two types of contribution:
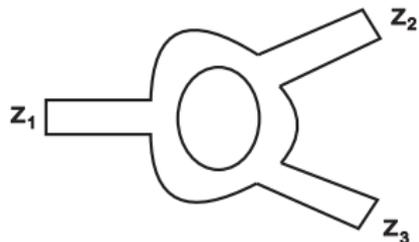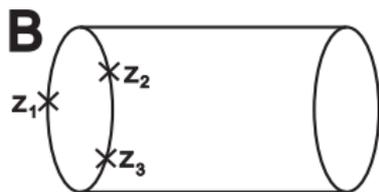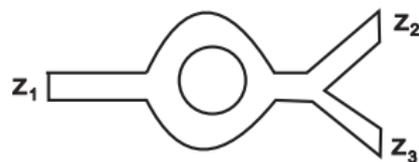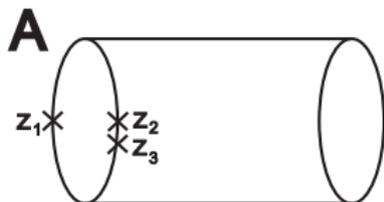
Yukawas: Wavefunction Renormalisation
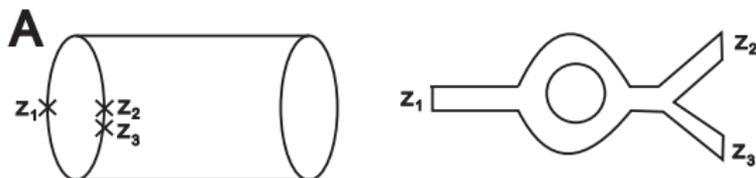
The amplitude has contributions from the IR ($t = \infty$) to the effective UV cutoff ($t \sim \frac{1}{M_W^2}$).

$$\mathcal{A} \sim \int_0^\infty \frac{dt}{t} \mathrm{Tr}_L(t^a t^b t^c \theta^2) \mathrm{Tr}_R(\theta^2) \prod_i \left(-2\sin\pi\theta_i\right) Z(t),$$

▶ Logarithmic running associated to wavefunction renormalisation.

▶ Result comes only from $\mathcal{N} = 2$ sector (θ^2 sector) - as for gauge thresholds.

▶ Running starts from super-stringy scale $M_W = RM_s$ - as for gauge thresholds.

Yukawas : Vertex Renormalisation
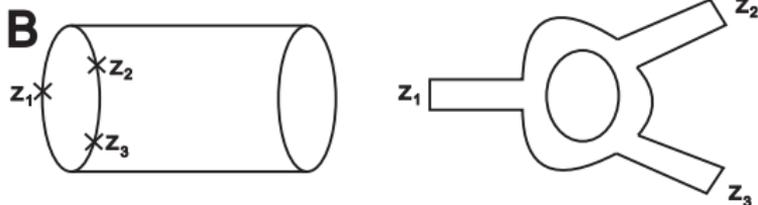
Amplitude becomes $\int \frac{dt}{t^3} \int dz_1 dz_2 dz_3 \, k_2 \cdot k_3 \frac{\vartheta_1(z_1 - z_2 + \theta_1)\eta^3}{\vartheta_1(z_1 - z_2)\vartheta_1(\theta_1)} \langle \prod_i e^{ik \cdot X(z_i)} \rangle$

with $k_2 \cdot k_3 = 0$ on-shell.

As $t \to \infty$,

$$\langle e^{ik \cdot X} \rangle \sim e^{-\alpha' k_i \cdot k_j (z_{ij} - \frac{z_{ij}^2}{t})}$$

This has a momentum pole as $z_{ij}, t \to \infty$ (infinite IR limit) which cancels $k_2 \cdot k_3$.

Contributions at finite t vanish (forbidden by non-renormalisation theorem), but there is a 1-loop vertex correction from the IR $t = \infty$ regime of the loop integral.

Yukawas : Vertex Renormalisation
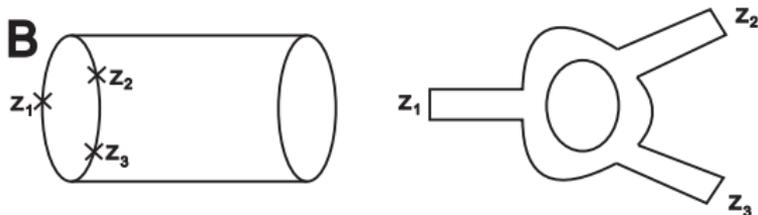
The vertex renormalisation is a (physical) IR effect associated to the 1PI action.

This can be verified by considering a 4-point amplitude.

In field theory it requires massless fields and arises from a term of the form (West 1991)

$$\int d^4x d^4\theta \, \frac{1}{\Box} D^2 g(\Phi) \rightarrow \int d^4x d^2\theta \, \frac{1}{\Box} \bar{D}^2 D^2 g(\Phi) \quad \rightarrow \quad \int d^4x d^2\theta \, \frac{\Box}{\Box} g(\Phi)$$

$$\rightarrow \quad \int d^4x d^2\theta \, g(\Phi).$$

Brane-to-Brane Susy Breaking
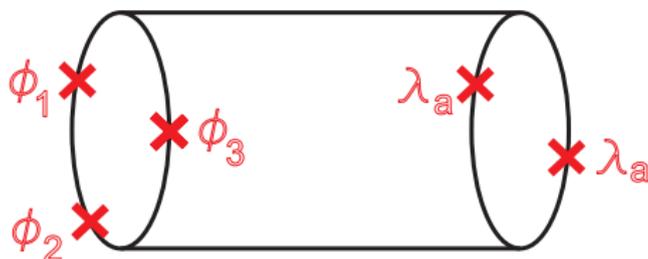
This is the correlator $\langle W \lambda \lambda \rangle$ where W is a field theory superpotential living on branes.

It gives gaugino masses induced by susy breaking vevs for ϕ_1, ϕ_2, ϕ_3.

Although this is an annulus diagram this in fact gives supergravity tree-level susy breaking (understand via closed string diagram).

Brane-to-Brane Susy Breaking

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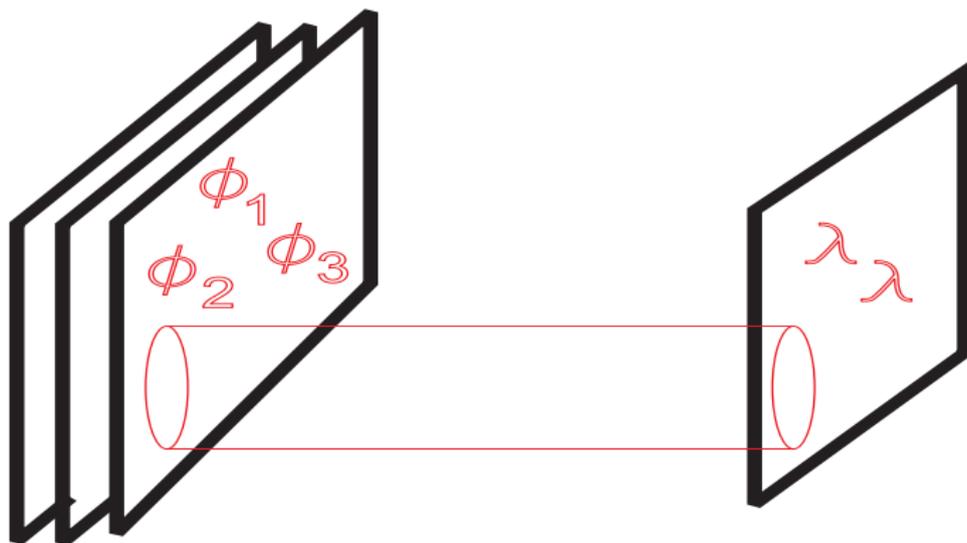
Flux-Induced Gaugino Masses

Tree-level flux induced gaugino masses studied on the disk by Billo, Ferro, Frau, Fucito, Lerda, Morales

Now come to target computation:
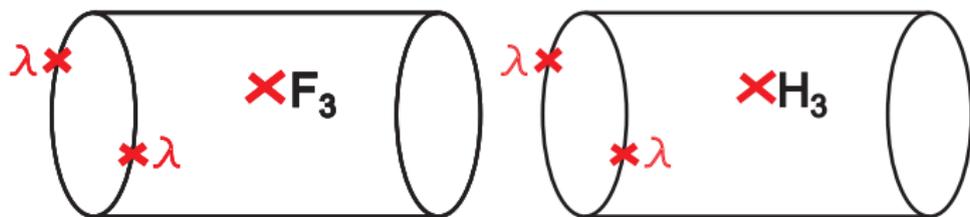
What are we looking for and how to interpret it?

Supergravity Expectations

$$m_{1/2,tree} = \frac{F^I \partial_I f_a}{2\text{Re}(f_a)}$$

$$
\begin{aligned}
m_{1/2,anomaly} &= -\frac{g^2}{16\pi^2}\left[\delta_{AM}\left(3T_G - T_R\right)m_{3/2} - \left(T_G - T_R\right)K_i F^i - \frac{2T_R}{d_R}F^i\partial_i\left(\ln\det Z\right)\right.\\
&\quad \left. + 2T_G F^I \partial_I \ln\left(\frac{1}{g_0^2}\right)\right].
\end{aligned}
$$

Consider this formula for 3-form flux backgrounds in the case of NS-NS and RR flux.

$$W = \int G_3 \wedge \Omega$$

for G_3 pure NS-NS or pure RR (G_3 is $(0,3) \pm (3,0)$).

Note that in worldsheet CFT

$$G_3 = F_3 - iSH_3 \rightarrow F_3 - \frac{i}{g_s}H_3$$

NS-NS flux

$$W = -iS \int H_3 \wedge \Omega$$

Tree mass $\qquad M_{\lambda, tree} = \dfrac{F^S}{2\text{Re}(S)} = \bar{m}_{3/2},$

Running couplings: $\qquad \dfrac{1}{g^2(\mu)} = \dfrac{1}{g_{tree}^2}\left(1 + \dfrac{g^2 b_a}{16\pi^2}\ln\left(\dfrac{M_W^2}{\mu^2}\right)\right)$.

Running masses $\qquad M_{\lambda, running} = \bar{m}_{3/2}\left(1 - \left(\dfrac{g^2 b_a}{16\pi^2}\right)\ln\left(\dfrac{M_W^2}{\mu^2}\right)\right)$.

Anomalous masses $\qquad M_{\lambda, anomaly} = \dfrac{g^2 b_a}{16\pi^2}\delta_{AM}\bar{m}_{3/2}$.

R-R flux

$$W = \int F_3 \wedge \Omega$$

Tree mass $\qquad M_{\lambda, tree} = \dfrac{F^S}{2\text{Re}(S)} = -\bar{m}_{3/2},$

Running couplings: $\qquad \dfrac{1}{g^2(\mu)} = \dfrac{1}{g_{tree}^2}\left(1 + \dfrac{g^2 b_a}{16\pi^2}\ln\left(\dfrac{M_W^2}{\mu^2}\right)\right)$.

Running masses $\qquad M_{\lambda, running} = -\bar{m}_{3/2}\left(1 - \left(\dfrac{g^2 b_a}{16\pi^2}\right)\ln\left(\dfrac{M_W^2}{\mu^2}\right)\right)$.

Anomalous masses $\qquad M_{\lambda, anomaly} = \dfrac{g^2 b_a}{16\pi^2}(\delta_{AM} - 2)\bar{m}_{3/2}$.

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String computation of $\langle H_3 \lambda \lambda \rangle$ or $\langle F_3 \lambda \lambda \rangle$ can give both running and anomalous mass terms.

Test formula by looking at ratio of anomalous mass and running mass.

$$\frac{M_{\lambda, running}}{M_{\lambda, anomaly}} = \begin{cases} -\frac{1}{\delta_{AM}} \ln\left(\frac{M_W^2}{\mu^2}\right) & \text{NSNS} \\ \frac{1}{\delta_{AM} - 2} \ln\left(\frac{M_W^2}{\mu^2}\right) & \text{RR} \end{cases}$$

Can check results independently via computations with NS-NS and RR flux vertex operators.

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Want

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2 w \langle \mathcal{V}_\lambda^{-1/2}(z_1) V_\lambda^{1/2}(z_2) \mathcal{V}_{H_3}^{0,0}(w) \rangle$$

Flux vertex operator in $(-1, -1)$ picture is

$$\mathcal{V}_B^{(-1,-1)} = H_{123} e^{-\phi(w) - \bar{\phi}(\bar{w})} \overline{X}^3(w, \bar{w}) \left(\overline{\psi^1}(w) \overline{\widetilde{\psi}^2}(\bar{w}) - \overline{\psi^2}(w) \overline{\widetilde{\psi}^1}(\bar{w}) \right) e^{ik \cdot X(w, \bar{w})}.$$

Flux vertex operator in $(0, 0)$ picture is

$$
\begin{aligned}
\mathcal{V}_B^{(0,0)} &= \frac{1}{4} H_{123} e^{ik \cdot X} \left[-\alpha' \overline{\psi^3} \left(\overline{\psi^1} \left(\overline{\partial} \overline{X}^2 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\widetilde{\psi}^2} \right) - \overline{\psi^2} \left(\overline{\partial} \widetilde{X}^1 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\widetilde{\psi}^1} \right) \right) \right. \\
&\quad - \alpha' \overline{\widetilde{\psi}^3} \left(\left(\partial \overline{X}^1 - \frac{i\alpha'}{2} \left(k \cdot \psi \right) \overline{\psi^1} \right) \overline{\widetilde{\psi}^2} - \left(\partial \overline{X}^2 - \frac{i\alpha'}{2} \left(k \cdot \psi \right) \overline{\psi^2} \right) \overline{\widetilde{\psi}^1} \right) \\
&\quad + \overline{X}^3 \left(\left(\partial \overline{X}^1 - \frac{i\alpha'}{2} \left(k \cdot \psi \right) \overline{\psi^1} \right) \left(\partial \overline{\widetilde{X}}^2 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\widetilde{\psi}^2} \right) \right. \\
&\quad \left. \left. - \left(\partial \overline{X}^2 - \frac{i\alpha'}{2} \left(k \cdot \psi \right) \overline{\psi^2} \right) \left(\partial \overline{\widetilde{X}}^1 - \frac{i\alpha'}{2} \left(k \cdot \tilde{\psi} \right) \overline{\widetilde{\psi}^1} \right) \right) \right].
\end{aligned}
$$

Two kinds of pole: can move flux vertex operators onto gaugini
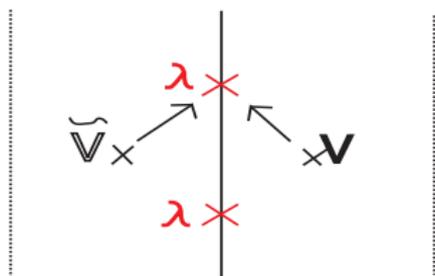
Move flux vertex operator onto gaugino to generate a momentum pole. There are two residual vertex operator integrals

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \rightarrow \int \frac{dt}{t^3} \int dz_1 dz_2 \rightarrow \int \frac{dt}{t}.$$

This gives a running mass

$$\mathcal{A} \sim \int \frac{dt}{t} \frac{-2k_2 \cdot k_3 - k_3 \cdot k_3}{2k_2 \cdot k_3 + k_3 \cdot k_3} Z(t)$$

Two kinds of pole: or move flux vertex operators onto boundary
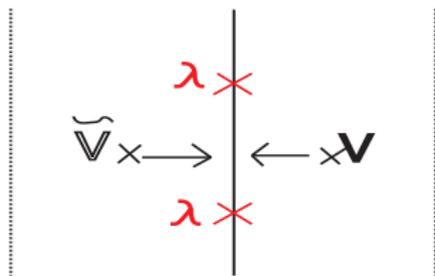
$$\mathcal{A} \sim \int \frac{dt}{t^3} \int dz_1 dz_2 d\mathrm{Im}(w) \frac{k_3 \cdot k_3}{k_3 \cdot k_3} \langle \partial_n X^{\bar{3}}(z_2) \left(X^3(iw_2) - X^3(1/2 + iw_2) \right) \rangle.$$

$$\langle \partial_n X^{\bar{3}}(z_2) \left(X^3(iw_2) - X^3(1/2 + iw_2) \right) \rangle \quad = \quad \frac{dZ_{winding}}{dt}$$

$$= \sum_{n,m} (n^2 + m^2) R^2 e^{-(n^2+m^2)R^2)\alpha' t} - \sum_{n,m} ((n+\tfrac{1}{2})^2 + (m+\tfrac{1}{2})^2) R^2) e^{-((n+\frac{1}{2})^2 + (m+\frac{1}{2})^2)R^2)\alpha' t}$$

Definitions and Motivation   Gauge Thresholds
Calculations   1-loop Yukawa Couplings
Conclusions   **Flux-induced Gaugino Masses**
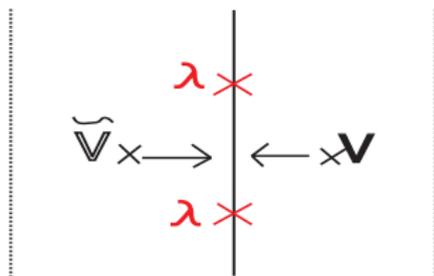
$$
\begin{aligned}
\mathcal{A} \quad &\sim \quad \frac{k_3 \cdot k_3}{k_3 \cdot k_3} \int \frac{dt}{t^3} \int dz_1 dz_2 d\mathrm{Im}(w) \frac{dZ_{winding}}{dt} \\
&\sim \quad \int_{t=0}^{\infty} \frac{dZ}{dt} \\
&= \quad Z(\infty) - Z(0) = Z(\infty).
\end{aligned}
$$

This is an anomalous mass term.

Note it arises from a sum purely over ultraviolet states (only non-zero winding number contributes to the sum), is only well-defined for a tadpole-cancelling theory, but is then determined entirely by the massless spectrum.

Overall amplitude in low-momentum limit is

$$\int \frac{dt}{t} \left(\frac{2k_2 \cdot k_3 + k_3 \cdot k_3}{-2k_2 \cdot k_3 - k_3 \cdot k_3} Z(t) + t \frac{k_3 \cdot k_3}{k_3 \cdot k_3} \frac{d}{dt} Z(t) \right)$$

Poles and zeros cancel and there is a definite answer in the on-shell limit $k_i \cdot k_j = 0$, $k_i^2 = 0$.

Anomalous and mass terms are opposite sign and same magnitude - precisely consistent with $\delta_{AM} = 1$.

Definitions and Motivation
Calculations
Conclusions

Gauge Thresholds
1-loop Yukawa Couplings
Flux-induced Gaugino Masses

RR Flux

Can do the same thing for RR flux. Vertex operator is

$$V_F^{(-1/2,-1/2)} = N_F g_s e^{-\phi/2-\tilde\phi/2} F_{mnp} \Theta(z) C \Gamma^{mnp} \tilde\Theta(\tilde z) e^{ik\cdot X},$$

Can compute

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{1/2}(z_1) \mathcal{V}_\lambda^{1/2}(z_2) \mathcal{V}_F^{-1/2,-1/2}(w,\bar w)\rangle$$

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{1/2}(z_1) \mathcal{V}_\lambda^{-1/2}(z_2) \mathcal{V}_F^{1/2,-1/2}(w,\bar w)\rangle$$

$$\int \frac{dt}{t^3} \int dz_1 dz_2 d^2w \langle \mathcal{V}_\lambda^{-1/2}(z_1) \mathcal{V}_\lambda^{-1/2}(z_2) \mathcal{V}_F^{1/2,1/2}(w,\bar w)\rangle$$

and evaluate correlators in zero momentum limit.

RR Flux

- ▶ With the RR flux, there are again both running and anomalous contributions to gaugino masses.

- ▶ However there is an off-shell ambiguity in fixing the relative sign and magnitude of these two terms.

  This is associated to the use of a 3-point function to study the amplitude.

- ▶ In principle this can be resolved by considering a 4-point function, where there is no need to continue off-shell, but this is calculationally prohibitive.

Conclusions

▶ We have studied 1-loop flux-induced gaugino masses on the worldsheet.

▶ Anomalous mass terms exist and take the form

$$
\begin{aligned}
M_a \sim \frac{m_{3/2}}{16\pi^2} \int dt \frac{d}{dt} Z(t) &= \frac{m_{3/2}}{16\pi^2} \left(Z(\infty) - Z(0) \right) \\
&= \frac{m_{3/2} b_a}{16\pi^2}.
\end{aligned}
$$

These appear as an explicit sum over UV states, but the result depends only on the massless spectrum.

▶ For NS-NS flux sign and magnitude of anomalous mass term agrees with BMP formula.

▶ RR flux case would require a 4-point computation to resolve off-shell ambiguities present in the 3-point function.